〔Article〕

Development by Exploiting Colonies: A Modified Nelson Model

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1. Introduction

Lewis (1954) has started the study of dual economies, especially the effects of the existence of a sector with virtually unlimited supply of labour on various economic consequences by specific policies. Based upon Mrs Ela Bhatt's philosophy (Bhatt (1995)), I have myself presented, in a series of papers Ekuni (2001a, 2001b, 2002a, and 2002b), some models with a duality between two sexes, men and women, and have shown how women may be able to contribute economic development through a simple device such as saving accounts for women protected from men's appropriation.

On the other hand, there is a well-known article by Nelson (1956) on poverty traps. This initiated a considerable amount of literature on many other types of poverty traps a country may encounter while struggling to take off to a smooth economic development. Nelson's original trap created by the interplay between the saving ratio and the population growth rate seems, after all, robust and inescapable. Then, an immediate question is how the British Empire and France were successful

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in jumping out of the Nelson's trap. Certainly, a natural answer is that these countries colonized many regions and exploited peoples as well as resources there. This natural answer, it seems, has not been formally presented in the literature. The purpose of this note is to provide a model of two countries *a la* Nelson embodying the above argument, and also simulation analyses to offer concrete numerical examples.

In Section2, Nelson's model is explained for a model of a single country, and then in Section 3, it is modified to include two countries. A couple of propositions are given in this Section 3. Section 4 explains simulation results. Some remarks are made in the final Section 5.

2. Nelson's Model

Here we give a brief account of Nelson's model in Nelson (1956). First the symbols are:

Y: national (or domestic) income;

L: labour force;

K: capital stock;

S: savings;

I: investment;

y: per capita national income, i.e., $y=Y/L$;

σ: the capital-output ratio;

$n(y)$: the rate of population growth or labour force;

$s(y)$: the saving ratio.

A more detailed symbol such as $K(t)$ means that variable for period t.

The following assumptions are made:

Assumption 1. The capital-output ratio σ is constant. (Technical progress may or may not increase this ratio. And yet, we assume the overall effect is neutral.)

Assumption 2. The rate $n(y)$ is negative when y is very small, and then increases, gets positive, attains the maximum, and finally starts to decrease.

Assumption 3. The ratio *s(y)* is negative when y is very small, and then increases and gets positive.

We in addition assume that capital stock lasts for ever with no depreciation, or that the saving and the investment are magnitudes net of depreciation. The dynamic change of our economy is described by the following equation.

$$\hat{y} \equiv \widehat{\left(\frac{Y}{L}\right)} = \hat{Y} - \hat{L} = \hat{K} - n(y) = \frac{I}{K} - n(y) = \frac{S}{K} - n(y) = \frac{s(y)\cdot Y}{K} - n(y) = \frac{s(y)}{\sigma} - n(y)$$

where a caret mark over a variable stands for the annual rate of change of that variable. More precisely, $\hat{x}(t)=(x(t+1)-x(t))/x(t)$, with t showing period t. When two curves *n(y)* and $s(y)/\sigma$ are like those shown in Fig.1, that is, they have two points in common, the lower equilibrium y^* turns out as the poverty trap, which is locally stable, and the higher equilibrium y^{**} is unstable.

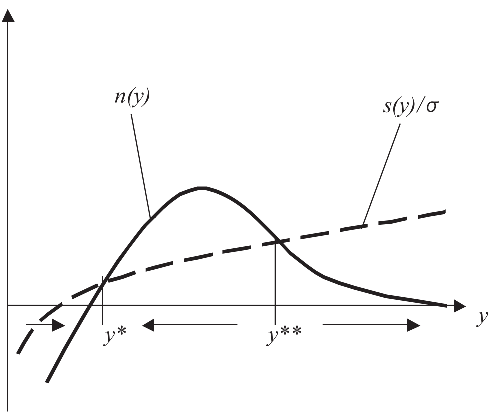

Fig.1 Nelson's Model

In simulation, we can use simply the normal variables, thus

$$Y(t)=K(t)/\sigma,\ \ y(t)\equiv Y(t)/L(t),\ \ I(t)=S(t)=s(y(t))\cdot Y(T),$$

$$K(t+1)=K(t)+I(t),\ \ \text{and}\ \ L(t+1)=L(T)\cdot(1+n(y(t))).$$

3. Our Two-Country Model with Colonization

Our model in this note is just a modification of Nelson's discrete model above by including two countries or regions. They are called a sovereign country and a colonized region. The symbols pertinent to these countries are shown with a subscript s and a c, respectively. For example, y_s is for the sovereign country, while y_c for the colonized area.

We continue to make the three assumptions in the previous section for individual regions, and now one more:

Assumption 4. A constant portion e of the *capital stock* in the colonized region is transferred to the sovereign country: forcibly or under the name of fair trade we do not care. This transfer is made instantly at the end of each period.

Then the dynamics of two regions become:

for the sovereign country

$$K_s(t+1)=K_s(t)+I_s(t)+e\cdot K_c(t),$$

and for the colonized region

$$K_c(t+1)=K_c(t)+I_c(t)-e\cdot K_c(t),$$

We can now state two propositions, each of which is so evident.

Proposition 1. When e=0, both regions cannot enter the phase of smooth development, when their initial state in below the higher equilibrium respectively.

Proposition 2. When e is sufficiently large, but not so large as to destroy the colonized region, the sovereign country may be able to enter the phase of smooth development.

4. Simulation Analyses

To materialize our intuition, it is desirable to present some simple numerical examples using a PC. Our simulation analyses have been conducted for the functions depicted in Fig.2, assuming both regions have the same $n(y)$ and $s(y)/\sigma$. Actually these functions and the constant σ are:

$$n(y) \equiv \begin{cases} 0.005(y-10) & \text{for } 0 \le y < 15 \\ -0.0002(y-15)+0.025 & \text{for } 15 \le \text{y} < 140, \\ 0 & \text{for } 140 \le \text{y}. \end{cases}$$

$$s(y) \equiv -0.4/(0.1y+1)+0.2 \qquad \text{for } 0 \le y \text{, and}$$

$$\sigma = 4.$$

There exist in fact three cross points, i.e., equilibrium levels, causing no problem. They are (0, −0.05), y*=(10, 0), and y**=(26.6, 0.0227) : the last equilibrium point y** shows approximate values. The initial values are: $K_s(0)$=80, $K_c(0)$=70, and $L_s(0)$=$L_c(0)$=1.

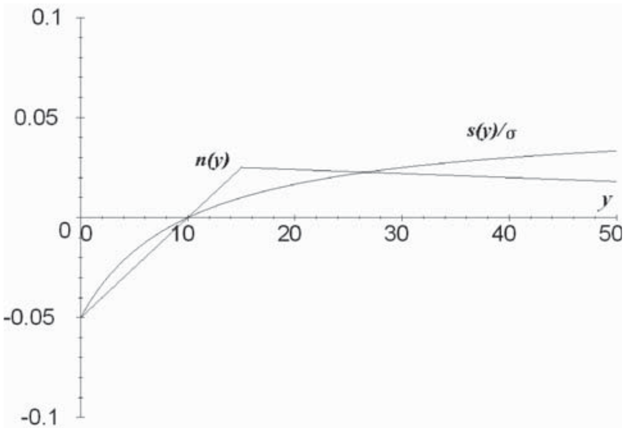

Fig.2 Functions in Simulation Analyses

Fig.3 presents the case with no transfer, i.e., e=0, confirming the existence of the poverty trap for both regions. In the figures, the curve below shows the annual magnitudes of per capital income of the sovereign country $y_s(t)$, while the top one of the colonized region, and calculation is made for 256 years.

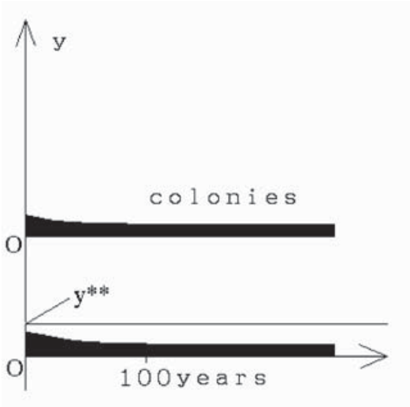

Fig.3 No Exploitation, e=0,

Then, Fig.4 gives the case where e=0.027. (Fig.4 also contains the numerical values of variables written out.) We can see that the sovereign country is able to escape the poverty trap by appropriating a fraction as low as 2.7% of the colony's capital stock. (We should note that this small 2.7% is surely a extremely heavy burden on the colony.) When σ=4, and e=0.027, it takes 50 years for the sovereign country cross the critical value y^{**}=26.6, after starting at y=20. (In our setting, the transfer ratio e=0.026 seems to be a point of bifurcation below which both countries are trapped in the poverty hole.) If we set e=0.03 with σ=4, the years necessary to go over the critical value get shortened to 33 years. If we choose σ=3.8, keeping e=0.027, it takes only 19 years to get into the smooth development phase. Fig.5 represents this case. Thus, the development paths are likely to be more sensitive to the changes in capital-output ratio.

Nelson Model(1956) :fmk[2007]

*Year	Capital	Labour	K/L	y
239	59933.1	132.308	452.98	113.25
240	62443.5	133.015	469.45	117.36
241	65075.4	133.618	487.03	121.76
242	67835.3	134.105	505.84	126.46
243	70729.9	134.468	526.00	131.50
244	73766.6	134.697	547.65	136.91
245	76952.8	134.780	570.95	142.74
246	80296.6	134.780	595.76	148.94
247	83806.2	134.780	621.80	155.45
248	87490.0	134.780	649.13	162.28
249	91356.7	134.780	677.82	169.45
250	95415.4	134.780	707.93	176.98
251	99675.9	134.780	739.54	184.89
252	104148.2	134.780	772.73	193.18
253	108843.1	134.780	807.56	201.89
254	113771.5	134.780	844.13	211.03
255	118945.4	134.780	882.51	220.63
256	124376.9	134.780	922.81	230.70
256	124376.9	134.780	922.81	230.70

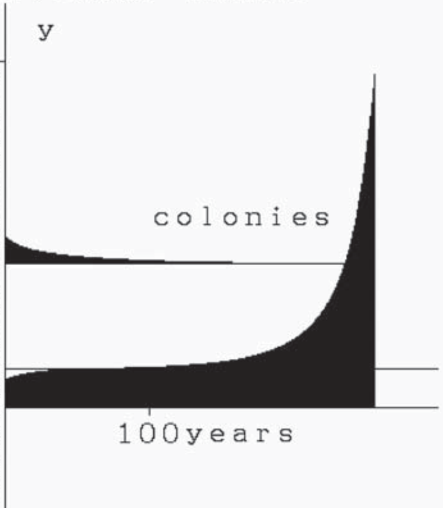

Fig.4 A Result of Simulation with Variable Values Shown

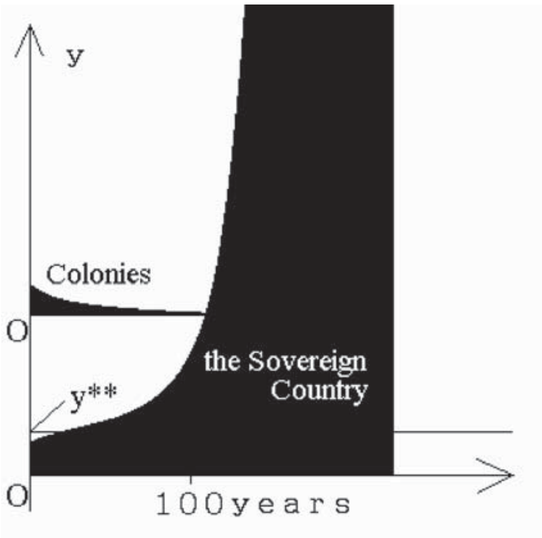

Fig.5 σ =0.38, e=0.027

5. Concluding Remarks

To conclude this memorandum, we write down three remarks, the second of which indicates our future research topics.

Remark 1. If we had made Assumption 4 as postulating the transfer of income *Y(t)* instead of capital stock *K(t)*, then the dynamics would become,

for the sovereign country,

$$\hat{y}_s = \frac{s_s(y)}{\sigma_s} - n_s(y) + e \cdot L_c \cdot y_c / (L_s \cdot y_s)$$

and for the colonized region

$$\hat{y}_c = \frac{s_c(y)}{\sigma_c} - n_c(y) - e$$

In this mechanism, it is not easy for the sovereign country gets out of the poverty trap. This is simply because the transfer merely shifts the present state *y(t)* to the right. Thus, unless the transfer is not large enough to go beyond the critical value y^{**}, the economy is destined to be pulled down to the trap.

Remark 2. The colonized regions not only provided raw materials to the Empire, but also they provided vast markets for the latter. To incorporate this double function of the colonies, we need to disaggregate our model to two- or three-sector model of development.

It may also be necessary to involve more than two countries, and to construct a multi-country model

Remark 3. In this note, we are not insisting that in the past the British Empire did succeed in going over the higher equilibrium because she exploited her colonies. What we have tried to tell the reader is that the existence of colonies was a great help for the Empire to skip beyond the pass, which was otherwise difficult to break through.

Appendix: The Pascal Programme for Simulation

```
(*****************************************************************)
(* Nelson Model of Poverty Traps by R Nelson (AER,[1956]). tf:2007-10   *)
(*****************************************************************)
program Nelson_C;      {=====  Turbo PASCAL  ver.3  =====}
{$C-}
{$i TPLIO.LIB}      {* graphics library by tf        *}
{$i window.lib}

const
    K0: real=80.0;    { capital at the beginning }
    K0C:real=70.0;    { C attached for colonies  }
    L0: real=1.0;     { labour  at the beginning }
    L0C:real=1.0;
    sigma:real = 4.0; { capital-output ratio     }
    IT: integer= 256; { repeated years           }
    e:  real=0.027;   { rate of exploitation     }
                      { sigma=4 -> e=0.026       }

var
    vmes:string[80];
    Y0,Y0C:       real;    { GDP; C = colonies      }
    KT,LT,KTC,LTC: real;   { at year T, K(t+1) etc  }
    y,yC:         real;    { per worker             }
    ch: char    ;
    i,j,PH,PV:integer;
    equi: real;

function pop_growth(y:real):real;
begin
```

```
 if y<15.0 then pop_growth := 0.005*(y-10.0)
 else if y<140.0 then pop_growth := -0.0002*(y-15.0)+0.025
 else pop_growth := 0.0
end;

function saving_ratio(y:real):real;
begin
 saving_ratio := ((-0.4)/(0.1*y+1.0))+0.2
end;

procedure frame;
var
ir,jr:real;
begin
 gline(300,300,600,300,6,0);
 gline(300,0,300,400,6,0);
 gline(400,300,400,305,3,0);

 equi:=26.6;   { equilibrium }
 PH:=300+300; PV:=300-round(equi);
 gline(300,PV,PH,PV,5,0);
 vmes:='１００ｙｅａｒｓ';
 gwrite(375,310,vmes,6,1);
 vmes:='ｙ';
 gwrite(320,30,vmes,7,1);
end;

begin
  w_make(0,4,37,18); (* text window *)
 gbegin;
```

```
frame;
  gline(0,0,639,16,10,2);    (* title  *)
  gline(0,383,639,399,12,2); (* notice・*)
   gLineStyle:=$FFFF;
  gline(0,60,299,60,7,0);
tcolor(3);
gotoxy(16,1);
write('Ｎｅｌｓｏｎ　Ｍｏｄｅｌ（１９５６）　： [mfmk[2007]');
tcolor(6);
gotoxy(7,25);
write('  ***   Normal Key = STOP/Restart, '
                 + ' Q or q key = Quit   ***   ');
gotoxy(1,3);
write('*Year   [32mCapital   [33mLabour    [mK/L   y');
tcolor(7);
 w_gotoxy(0,0);  (* text window *)

for i:=1 to IT do begin     (* iteration for IT years *)
 write(I:4,' [32m',K0:10:1,' [33m',L0:8:3,' [m',K0/L0:8:2);
 write(' [35m',(K0/sigma)/L0:7:2,' [m');

w_str[0]:=chr(0);  (* text window *)
w_writeln(w_str);  (* scroll *)
Y0 := K0/sigma;
Y0C:= K0C/sigma;

y  := Y0/L0;
yC := Y0C/L0C;
KT := (1.0+(saving_ratio(y)/sigma))*K0;
KTC:= (1.0+(saving_ratio(yC)/sigma))*K0C;
```

```
 KT := KT + e*KTC;  (* out of capital stock, confiscated *)
 KTC:= KTC- e*KTC;

 LT := (1.0+pop_growth(y))*L0;
 LTC:= (1.0+pop_growth(yC))*L0C;
  PH:=300+i; PV:=300-(round(y));
  gline(PH,300,PH,PV,4,0);
   PV:=200-(round(yC));
   gline(PH,200,PH,PV,5,0);
 K0 := KT;
 K0C:= KTC;
 L0 := LT;
 L0C:= LTC;

      if KeyPressed then begin  (* when a special key pressed *)
         Read(KBD,ch);
         if (UpCase(ch)='Q') then begin (* quit when 'q' is pressed. *)
         gend0(55,22);
         halt;
         end;
         Read(Kbd,ch);
         if (UpCase(ch)='Q') then begin
         gend0(30,12);
         halt;
         end;
      end;
 end;(* iteration *)
 gend0(55,22);
end.
```

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